

The quark-photon vertex and meson electromagnetic form factors

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The ladder Bethe–Salpeter solution for the dressed photon-quark vertex is used to study the low-momentum behavior of the pion electromagnetic and the $\gamma^*\pi^0\gamma$ transition form factors. With model parameters previously fixed by light meson masses and decay constants, the low-momentum slope of both form factors is in excellent agreement with the data. In comparison, the often-used Ball–Chiu Ansatz for the vertex is found to be deficient; less than half of the obtained r_π^2 is generated by that Ansatz while the remainder of the charge radius could be attributed to the tail of the ρ resonance.

1. DYSON–SCHWINGER EQUATIONS

The Dyson–Schwinger equations [DSEs] form a useful tool for nonperturbative QCD modeling of hadrons and their interactions. They have been successfully applied to calculate properties of light vector and pseudoscalar mesons [1,2], as described elsewhere in these proceedings [3]. The dressed-quark propagator, as obtained from its DSE, together with the Bethe–Salpeter amplitude as obtained from the Bethe–Salpeter equation [BSE] for $q\bar{q}$ bound states, form essential ingredients for calculations of meson couplings and form factors [4]. To describe electromagnetic interactions of hadrons, we also need the nonperturbatively dressed quark-photon vertex. Here, we use a solution of the DSE for the quark-photon vertex under the same truncation as used in Refs. [1–3] for light mesons.

The DSE for the renormalized dressed-quark propagator in Euclidean space is

$$S(p)^{-1} = Z_2 i\gamma \cdot p + Z_4 m(\mu) + Z_1 \int \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S(q) \Gamma_\nu^a(q, p), \quad (1)$$

where $D_{\mu\nu}(k)$ is the dressed-gluon propagator and $\Gamma_\nu^a(q; p)$ the dressed-quark-gluon vertex. The solution of Eq. (1) has the form $S(p)^{-1} = i\gamma \cdot p A(p^2) + B(p^2)$ and is renormalized at spacelike μ^2 according to $A(\mu^2) = 1$ and $B(\mu^2) = m(\mu)$ with $m(\mu)$ the current quark mass.

The DSE for the quark-photon vertex $\tilde{\Gamma}_\mu(p; Q) = \hat{Q} \Gamma_\mu(p; Q)$ is the inhomogeneous BSE

$$\tilde{\Gamma}_\mu(p; Q) = Z_2 \hat{Q} \gamma_\mu + \int^\Lambda \frac{d^4q}{(2\pi)^4} K(p, q; Q) S(q + Q/2) \tilde{\Gamma}_\mu(q; Q) S(q - Q/2), \quad (2)$$

where \hat{Q} is the charge operator. The kernel K is the renormalized, amputated $\bar{q}q$ scattering kernel that is irreducible with respect to a pair of $\bar{q}q$ lines. Solutions of the homogeneous

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version of Eq. (2) define the vector meson bound states at $Q^2 = -m^2$. It follows that $\tilde{\Gamma}_\mu(p; Q)$ has poles at those locations.

We use a ladder truncation for the BSE in conjunction with a rainbow truncation $\Gamma_\nu^a(q, p) \rightarrow \gamma_\nu \lambda^a / 2$ for the quark DSE. Both the vector Ward–Takahashi identity [WTI] for the quark-photon vertex and the axial-vector WTI are preserved in this truncation. This ensures both current conservation and the existence of massless pseudoscalar mesons if chiral symmetry is broken dynamically: pions are Goldstone bosons [1].

The details of the model can be found in Refs. [2,3]. It leads to chiral symmetry breaking and confinement; furthermore, at large momenta, our effective interaction reduces to the perturbative running coupling and thus preserves the one-loop renormalization group behavior of QCD and reproduces perturbative results in the ultraviolet region. The model gives a good description of the π , ρ , K , K^* and ϕ masses and decay constants [2,3].

2. THE DRESSED QUARK-PHOTON VERTEX

The general form of the quark-photon vertex $\Gamma_\mu(q; Q)$ can be decomposed into twelve independent Lorentz covariants. Four of these covariants, representing the longitudinal components, are uniquely determined by the vector WTI

$$i Q_\mu \Gamma_\mu(p; Q) = S^{-1}(p + Q/2) - S^{-1}(p - Q/2). \quad (3)$$

The eight transverse components of $\Gamma_\mu(p; Q)$ are not constrained by the WTI, except at $Q = 0$, where the WTI reduces to $i \Gamma_\mu(p; 0) = \partial S(p)^{-1} / \partial p_\mu$.

It is obvious from Eq. (3) that the bare vertex γ_μ is a bad approximation if the quark self-energy is momentum dependent as is realistically the case. For QCD modeling of electromagnetic coupling to hadrons, it has been common practice [5,6] to avoid a numerical study of the quark-photon vertex, and use the so-called Ball–Chiu [BC] Ansatz [7], which expresses the vertex in terms of the dressed-quark propagator functions A and B

$$\Gamma_\mu^{BC}(p; Q) = \gamma_\mu \frac{A(p_+^2) + A(p_-^2)}{2} + 2(\gamma \cdot p) p_\mu \frac{A(p_+^2) - A(p_-^2)}{p_+^2 - p_-^2} - 2i p_\mu \frac{B(p_+^2) - B(p_-^2)}{p_+^2 - p_-^2}, \quad (4)$$

where $p_\pm = p \pm Q/2$. This satisfies the WTI, Eq.(3), transforms correctly under CPT, and has the correct perturbative limit γ_μ in the extreme ultraviolet. The longitudinal components of Γ_μ^{BC} are exact, but the transverse components are correct only at $Q = 0$. In particular, Γ_μ^{BC} does not have the vector meson poles. This should be of little concern for form factors at large spacelike Q^2 ; however, for $Q^2 \approx 0$ the situation is less clear.

Our numerical solution of Eq. (2) shows clearly the vector meson pole in all eight transverse amplitudes. The solution for the four longitudinal amplitudes agrees perfectly with the BC Ansatz, as required by the WTI. Our transverse solution agrees with the BC Ansatz only at spacelike asymptotic momenta. At low Q^2 it departs significantly from this Ansatz; although there is necessarily agreement at the point $Q = 0$, the Q -dependence of the DSE solution is much larger than that of the BC Ansatz. Near the ρ pole, the quark-photon vertex behaves like

$$\tilde{\Gamma}_\mu(p; Q) \simeq \frac{\Gamma_\mu^\rho(p; Q) m_\rho^2 / g_\rho}{Q^2 + m_\rho^2}, \quad (5)$$

where the $\rho - \gamma$ coupling strength m_ρ^2/g_ρ associated with the $\rho \rightarrow e^+ e^-$ decay is well reproduced by the present model [2,3]. There is no unique decomposition of the vertex into resonant and non-resonant terms away from the pole, but over a limited interval near $Q^2 \approx 0$ the difference between the DSE solution and the BC Ansatz can be approximated by Eq. (5) with $m_\rho^2 \rightarrow -Q^2$ in the numerator, which one can call the tail of the ρ resonance. However, the DSE solution for the vertex is the appropriate representation containing both the resonant and non-resonant parts of the vertex.

3. MESON FORM FACTORS

In the impulse approximation, both the pion charge form factor and the $\gamma^* \pi \gamma$ transition form factor are described by a triangle diagram, with one or two pion Bethe-Salpeter amplitudes, one or two quark-photon vertices, and three dressed-quark propagators. We obtain these elements from the appropriate BSE and DSE, within the same model. We compare the form factor results using: 1) a bare vertex; 2) the BC Ansatz; 3) the DSE solution for the quark-photon vertex. Note that the WTI ensures electromagnetic current conservation in both 2) and 3), but not in approximation 1), which violates the WTI.

The impulse approximation for the pion form factor gives

$$F_\pi(Q^2)P_\nu = N_c \int \frac{d^4q}{(2\pi)^4} \text{Tr} [\Gamma_\pi(k_+; -P_+) S(q_{+-}) i\Gamma_\nu(q_+; Q) S(q_{++}) \Gamma_\pi(k_-; P_-) S(q_-)] , \quad (6)$$

where Q is the photon momentum, $P_\pm = P \pm Q/2$, $q_\pm = q \pm P/2$, $q_{\pm\pm} = q_\pm \pm Q/2$ and $k_\pm = q \pm Q/4$. It is evident that the Q^2 dependence of F_π comes from both the quark substructure of the pion and the Q -dependence of the quark-photon vertex. Due to Eq. (5), $F_\pi(Q^2)$ will exhibit a resonance peak at timelike momenta Q^2 near $-m_\rho^2$.

A long-standing issue in hadronic physics is the question of the extent to which $F_\pi(Q^2)$ at low spacelike Q^2 can still be described by the ρ resonance mechanism. This is an essential element of vector meson dominance (VMD), which leads to [8]

$$F_\pi^{VMD}(Q^2) = 1 - \frac{g_{\rho\pi\pi} Q^2}{g_\rho (Q^2 + m_\rho^2)} . \quad (7)$$

The first term arises from the non-resonant photon coupling to a point pion; the only Q^2 dependence in VMD comes from the resonant mechanism with the produced ρ having a point coupling to the pion. The pion charge radius $r_\pi^2 = -6F'_\pi(0)$ thus becomes $6g_{\rho\pi\pi}/(m_\rho^2 g_\rho) \sim 0.48 \text{ fm}^2$ which compares favorably with the experimental value 0.44 fm^2 .

In Fig. 1(a) we show our results from Eq. (6). Clearly a bare vertex is incorrect: current conservation, which ensures $F_\pi(0) = 1$, is violated. Use of the BC Ansatz conserves the current; but the resulting $r_\pi^2 = 0.18 \text{ fm}^2$ is less than half the experimental value and the curve misses the data completely. The DSE solution for the vertex agrees very well with the data and produces $r_\pi^2 = 0.45 \text{ fm}^2$, *without fine tuning the model parameters*: the parameters are completely fixed in Refs. [2,3]. This indicates that as much as half of r_π^2 can be attributed to a reasonable extrapolation of the ρ resonance mechanism. On the other hand, the strict VMD picture is too simple; at least 40% of r_π^2 arises from the non-resonant photon coupling to the quark substructure of the pion.

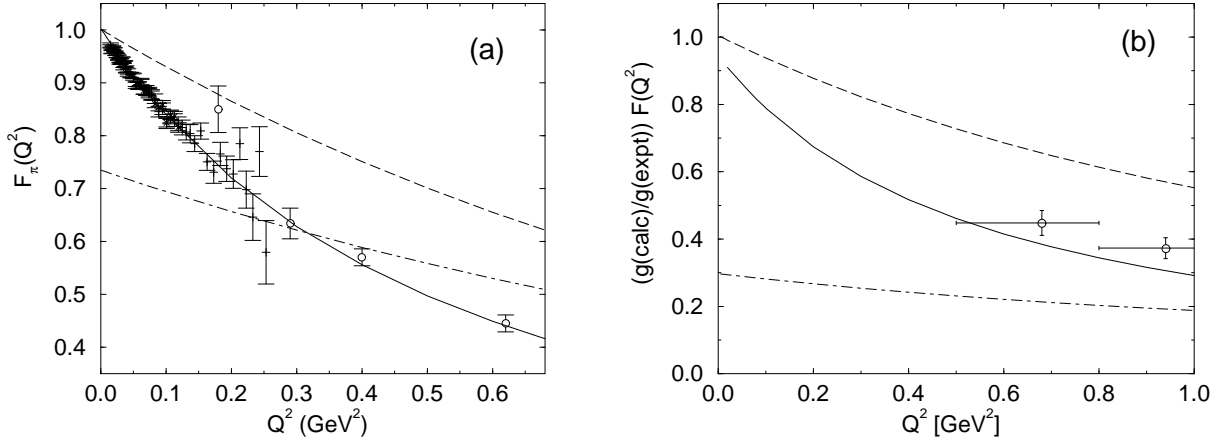


Figure 1. The pion form factor (a) and $\gamma^*\pi\gamma$ transition form factor (b) using a bare vertex (dot-dash), the BC Ansatz (dashed) and our numerical solution of Eq. (2) (solid).

The impulse approximation for the $\gamma^*\pi\gamma$ vertex with γ^* momentum Q is

$$\begin{aligned} \Lambda_{\mu\nu}(P, Q) &= i \frac{\alpha}{\pi f_\pi} \epsilon_{\mu\nu\alpha\beta} P_\alpha Q_\beta g_{\pi\gamma\gamma} F(Q^2) \\ &= \frac{N_c}{3} \int \frac{d^4k}{(2\pi)^4} \text{Tr} [S(q') i\Gamma_\nu(k'; Q) S(q'') i\Gamma_\mu(k''; -P - Q) S(q''') \Gamma_\pi(k; P)] . \end{aligned} \quad (8)$$

where the momenta follow from momentum conservation. In the chiral limit, the value at $Q^2 = 0$, corresponding to the decay $\pi^0 \rightarrow \gamma\gamma$, is given by the axial anomaly and its value $g_{\pi\gamma\gamma}^0 = 1/2$ is a direct consequence of only gauge invariance and chiral symmetry; this value corresponds well with the experimental width of 7.7 eV. In Fig. 1(b) we show our results, normalized to the experimental $g_{\pi\gamma\gamma}$. A bare vertex does not reproduce the anomaly since it violates WTIs. Both the BC Ansatz and the DSE vertex solution reproduce the anomaly value, but the BC Ansatz overestimates the form factor at small but nonzero spacelike momenta and gives an interaction radius $r^2 = 0.13 \text{ fm}^2$, compared to the experimental value $r^2 \sim 0.42 \text{ fm}^2$ [9]. The vertex DSE solution gives results remarkably close to the data and $r^2 = 0.40 \text{ fm}^2$, again indicating that the BC Ansatz underestimates the Q^2 dependence of the form factor, related to the absence of a ρ resonance.

REFERENCES

1. P. Maris and C.D. Roberts, Phys. Rev. **C56** (1997) 3369.
2. P. Maris and P.C. Tandy, nucl-th/9905056 (1999).
3. P. Maris, nucl-th/9908044 (1999), these proceedings.
4. P.C. Tandy, Prog. Part. Nucl. Phys. **39**(1997), 117.
5. P. Maris and C.D. Roberts, Phys. Rev. **C58** (1998) 3659.
6. P.C. Tandy, Fizika **B8** (1999) 295.
7. J.S. Ball and T.-W. Chiu, Phys. Rev. **D22** (1980) 2542.
8. H. O'Connell, B. Pearce, A. Thomas and A. Williams, Phys. Lett. **B354** (1995) 14.
9. H.-J. Behrend, *et al.* (CELLO Collaboration), Z. Phys. **C49** (1991) 401.